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Periodic Sequences that Facilitate Data Set Spectrum Measurements

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It is often desirable to measure the spectrum of the modulated baseband pulse of a quadrature amplitude modulated data set. The usual method, which is not very satisfactory, is to send a pseudorandom data sequence into the data set and measure the transmitted signal with a spectrum analyzer. Here we present a new technique, based on the properties of so-called "perfect sequences," which facilitates stable and accurate spectrum measurements.

I. INTRODUCTION

The modulated pulse spectrum (MPS), defined below, of a data set's signal is a basic characteristic of the data set that is of interest, for example, in determining how much bandwidth the signal requires. One can usually calculate an MPS, but it is often necessary to confirm the calculation by experiment. The usual practice in such experiments is to apply a binary pseudorandom sequence (also called maximum-length sequence[1]) to the input of a data set and measure the resulting output signal with a spectrum analyzer. This technique usually leads to a picture that shows the MPS approximately, but not exactly and not in a repeatable manner; the reasons are given in Section II.

The output of the spectrum analyzer is usually not constant at any one frequency setting, and the relative level at each frequency is not proportional to the MPS. An example is shown on Fig. 1.

If a data set modulator is a member of the quadrature amplitude modulation (QAM) class, which includes the differential phase shift keyed modulators, then there exist easily generated periodic constellation point sequences whose resulting signals have line spectra with amplitudes that are proportional to the modulator's MPS, and thus facilitate accurate experimental measurement of the MPS. The derivation and application of these sequences are the subject of this paper.

The next section establishes the properties required of the constel-

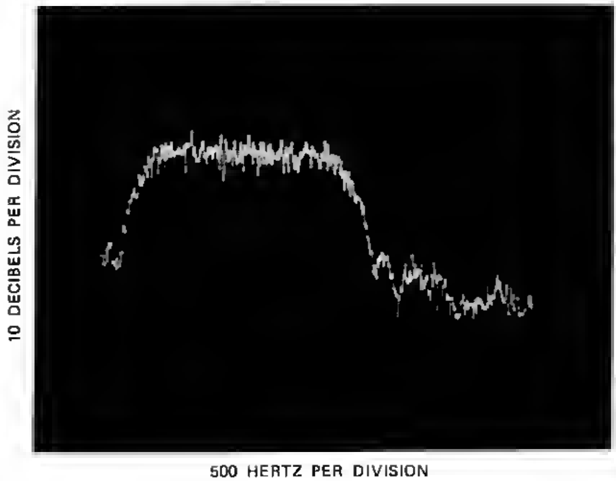

Fig. 1—Measured spectrum of a data set using the usual pseudorandom data method.

lation point sequences. Section III shows how the desired sequences can be derived from pseudorandom sequences and presents an example with experimental verification. Section IV presents another example and Section V contains the concluding remarks.

II. DATA SET SIGNALS

A QAM data set generates a signal that is often written as the real part of a complex signal, say $z(t)$, given by

$$z(t) = \sum_n C_n y(t - nT)e^{j\omega_c t}, \tag{1}$$

where ω_c is the carrier frequency, T is the intersymbol time, and $y(t)$ is a complex pulse waveform.[2,3] Each C_n is a complex constant selected from a finite set of allowable values by an algorithm that operates on the binary data to be transmitted (the input data stream). In general, several consecutive input bits determine each C_n. For example, the allowable real and imaginary parts of C_n may be ± 1 and ± 3, which gives 16 possible values (constellation points).

The real part of $z(t)$ is often passed through an analog filter, perhaps an equalizer or a low-pass filter, before being sent to the channel. Thus, if $f(t)$ is the impulse response of the filter, then the output signal is

$$s(t) = \mathrm{Re}\{s_0(t)\}, \tag{2}$$

where

$$s_0(t) = z(t) * f(t). \tag{3}$$

If the data set uses an analog modulator, then $y(t)$ is a real baseband pulse.

If the modulator is one that generates a step approximation to the corresponding continuous signal (digital implementation with D/A converter), then

$$y(t) = e^{-j\omega_c t} \sum_k h_k e^{jk\omega_c \tau} p(t - k\tau), \tag{4}$$

where the h_k are samples of a corresponding real baseband pulse, τ is the step width, and

$$p(t) = \begin{cases} 1, |t| < \tau/2 \\ ½, |t| = \tau/2 \\ 0, |0| > \tau/2. \end{cases} \tag{5}$$

We assume T/τ is an integer.

The Fourier transform of $s_0(t)$ is

$$S_0(\omega) = F(\omega) Y(\omega - \omega_c) C(\omega - \omega_c), \tag{6}$$

where $F(\omega)$ and $Y(\omega)$ are the Fourier transforms of $f(t)$ and $y(t)$ and

$$C(\omega) = \sum_n C_n e^{-jnT\omega}. \tag{7}$$

In the step approximation case,

$$Y(\omega - \omega_c) = P(\omega) H(\omega - \omega_c), \tag{8}$$

where

$$P(\omega) = \frac{\tau \sin(\omega\tau/2)}{(\omega\tau/2)} \tag{9}$$

and

$$H(\omega) = \sum_k h_k e^{-jk\tau\omega}. \tag{10}$$

We define the modulated pulse spectrum (MPS), for either case, to be

$$B(\omega) = F(\omega) Y(\omega - \omega_c). \tag{11}$$

Then

$$S_0(\omega) = B(\omega) C(\omega - \omega_c). \tag{12}$$

The Fourier transform of $s_0(t)$ is the product of two parts, the MPS and a function of the transmitted constellation sequence. The MPS is

closely related to the spectral density function of the signal, under suitable assumptions about the statistics of random C_n. For an example, see Ref. 4. When characterizing a data set signal, it is important to know the magnitude of the MPS and to be able to measure it.

If $\{C_n\}$ is a periodic sequence of period M, then $C(\omega)$ becomes

$$C(\omega) = A(\omega) \sum_{m=-\infty}^{\infty} \omega_0 \delta(\omega - m\omega_0), \tag{13}$$

where

$$\omega_0 = 2\pi/MT \tag{14}$$

and

$$A(\omega) = \sum_{n=0}^{M-1} C_n e^{-jnT\omega}. \tag{15}$$

Thus $s(t)$ consists of tones at frequencies $\omega_c + m\omega_0$ and levels given by $(1/MT)|B(\omega_c + m\omega_0)A(m\omega_0)|$. Therefore, if points along $|B(\omega)|$ are to be generated, one needs to select $\{C_n\}$ so that $|A(m\omega_0)|$ is a constant for all m. Notice that the periodic set of $A_m = A(m\omega_0)$ is the discrete Fourier transform (DFT) of $\{C_n\}_{n=0}^{M-1}$.

If the input binary data sequence is periodic and the usual types of scramblers and coders are used in the data set, then the resulting C_n sequence will be periodic. However, the corresponding A_m seldom have equal magnitude. That is one reason a spectrum analyzer gives a measurement of the sort shown in Fig. 1. The other reason is that the period, M, is usually so large that several of the resulting tone frequencies are in the narrow passband of the spectrum analyzer at the same time. The envelope of such a signal is modulated, giving an unstable spectrum analyzer output.

In the next section, we show how to overcome these problems.

III. PERFECT SEQUENCES

The autocorrelation function of a periodic sequence $\{C_n\}$, with period M, is:

$$R_c(k) = \sum_{n=0}^{M-1} C_{k+n} C_n^*. \tag{16}$$

If

$$R_c(k) = \begin{cases} \alpha_1, & k = 0 \text{ modulo } M \\ \alpha_2, & \text{otherwise}, \end{cases} \tag{17}$$

where α_1 and α_2 are constants, then the sequence $\{C_n\}$ is called a *perfect sequence.*[5]

One can show that, if

$$A_m = \sum_{n=0}^{M-1} C_n e^{-j2\pi nm/M}, \tag{18}$$

then

$$|A_m|^2 = \sum_{n=0}^{M-1} R_c(n) e^{-j2\pi nm/M}. \tag{19}$$

If $\{C_n\}$ is a perfect sequence, then

$$|A_m|^2 = \begin{cases} \alpha_1 + (M-1)\alpha_2, & m = 0 \text{ modulo } M \\ \alpha_1 - \alpha_2, & m \neq 0 \text{ modulo } M. \end{cases} \tag{20}$$

Thus the desired property of equal magnitude A_m can almost be achieved if $\{C_n\}$ is a perfect sequence (the exception is A_0). We next show how to construct perfect C_n sequences.

There are many perfect binary sequences.[6] Two are:

11011100010

1101000001000

One can also show that pseudorandom binary sequences are perfect sequences. An example is the sequence generated by

$$X_n = X_{n-3} \oplus X_{n-5}, \tag{21}$$

where $\oplus$ denotes modulo 2 addition. The period of this sequence is 31.

If X_n is a pseudorandom sequence of period M, then its autocorrelation function has two values:

$$R_x(k) = \begin{cases} \dfrac{M+1}{2}, & k = 0 \text{ modulo } M \\ \dfrac{M+1}{4}, & \text{otherwise.} \end{cases} \tag{22}$$

A perfect C_n sequence could be generated by simply using $C_n = X_n$. However, most QAM data sets do not include the value zero in their constellation. Thus, it is necessary to apply a linear transformation to a perfect binary sequence, $\{X_n\}$, to obtain a perfect sequence $\{C_n\}$ of allowable values.

Suppose α and β are two complex constants and $C_n = \alpha X_n + \beta$, then the autocorrelation function of $\{C_n\}$ is

$$\begin{aligned} R_c(k) &= \alpha\alpha^* R_x(k) + (\alpha^*\beta + \alpha\beta^*) \sum_{n=0}^{M-1} X_n + M\beta\beta^* \\ &= \alpha\alpha^* R_x(k) + (\alpha^*\beta + \alpha\beta^*) \frac{M+1}{2} + M\beta\beta^*. \end{aligned} \tag{23}$$

If $R_x(k)$ has the two levels given by (22), then

$$R_c(k) = \begin{cases} \alpha\alpha^* \dfrac{M+1}{2} + (\alpha^*\beta + \alpha\beta^*) \dfrac{M+1}{2} + M\beta\beta^*, & k = 0 \\ \\ \alpha\alpha^* \dfrac{M+1}{4} + (\alpha^*\beta + \alpha\beta^*) \dfrac{M+1}{2} + M\beta\beta^*, & k \neq 0. \end{cases} \tag{24}$$

Suppose, for example, we want each C_n to have one of two of the 16 possible values given in the 16-point constellation example above, say, $-3 - j3$ and $3 + j3$. Then we can choose $\alpha = 6 + j6$ and $\beta = -3 - j3$. The corresponding values of A_m, with $M = 31$, are

$$|A_m|^2 = \begin{cases} 18, & m = 0 \\ \\ 576, & m \neq 0. \end{cases} \tag{25}$$

Consider a data set with a carrier frequency of 1800 Hz and a symbol rate of 2400 baud. If the C_n sequence just described modulates the data set, the output signal will consist of tones at frequencies of

$$1800 + \frac{2400}{31} m \text{ Hz}$$

and the carrier level (corresponding to A_0) will be about 15 dB below the level of its two neighboring tones.

This example has been simulated by V. B. Lawrence using an experimental modem. The resulting measured spectrum is illustrated in Fig. 2. This can be compared to the calculated MPS for that data set,

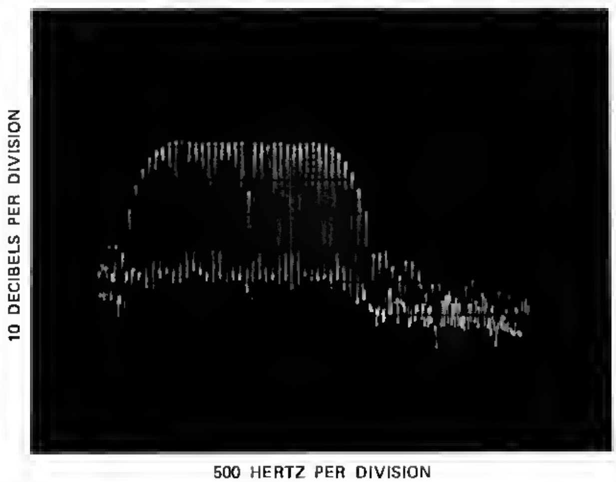

Fig. 2—Measured spectrum of a data set using the proposed method.

shown on Fig. 3. Notice that the carrier level on Fig. 2 is about 15 dB down from the level of its neighbors.

This technique can be applied to other types of QAM data sets by selecting binary sequences with desired periods and by choosing suitable values for the constants α and β.

IV. ANOTHER EXAMPLE

With some types of QAM data sets, it is feasible to find an input data sequence that will cause the data set to generate a perfect C_n sequence. An example is described below.

A differential PCM data set is on the market that uses unity-valued $C_n = e^{j\theta_n}$ and encodes consecutive pairs of bits (dibits) according to Table I.

One can cause the data set to generate a perfect sequence of C_n with the values 1 and -1 by an appropriate dibit sequence. To use a simple example, start with the 7-bit pseudorandom sequence 1110010. The corresponding periodic C_n sequence is:

$$1, 1, 1, -1, -1, 1, -1.$$

The periodic sequence of phase shifts must be

$$\pi, 0, 0, \pi, 0, \pi, \pi.$$

From the table, the corresponding periodic input data sequence is 11 10 10 11 10 11 11.

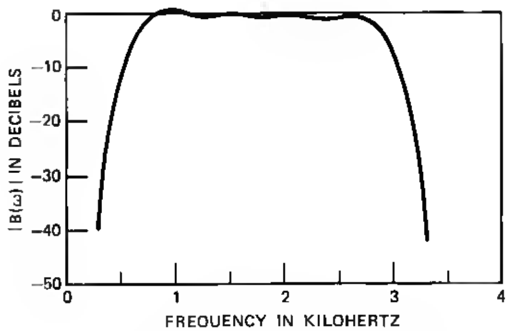

Fig. 3—Computed MPS of example data set.

Table I

Dibit	$\theta_n - \theta_{n-1}$
10	0
01	$\pi/2$
11	π
00	$-\pi/2$

If this periodic sequence were the input to the data set, the bits would be grouped by the data set into the desired dibit sequence or the following sequence, depending upon initial conditions:

$$11 \quad 11 \quad 01 \quad 01 \quad 11 \quad 01 \quad 11.$$

This sequence will produce a C_n sequence of period 28 that is not a perfect sequence. The spectrum of the line signal would be easily distinguished from the desired spectrum.

V. CONCLUSIONS

We have presented a simple method of generating a constellation point sequence such that, when it is applied to a QAM data set, the peaks (or envelope) of the resulting spectrum will be very nearly the MPS. The procedure is twofold: (*i*) select a period, M, small enough that the resulting tones can be separated by the spectrum analyzer and (*ii*) force $\{C_n\}$ to be a perfect sequence. The advantage of the method is that the spectrum can be accurately and repeatably measured with a spectrum analyzer. The accuracy is such that the observed spectrum will represent the MPS to within a few tenths of a decibel if the modulator is working properly. Modulator defects that would be obscured by the usual MPS estimation method are clearly evident when the proposed method is used.

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